

GRADES 9/10

EALR 1: The student understands and applies the concepts and procedures of mathematics.

Component 1.1: Understand and apply concepts and procedures from number sense.

Number and numeration

1.1.1 Understand and apply scientific notation. W

- Read and use scientific and exponential notation. [MC, RL]
- Identify a real-life situation to match a particular number written in scientific or exponential notation and justify the answer. [MC, RL]
- Use scientific or exponential notation to simplify a problem. [RL, MC]
- Illustrate the meaning of scientific notation using pictures, diagrams, or numbers. [CU]
- Read and translate numbers represented in scientific notation from calculators and other technology, tables, and charts.

1.1.4 Apply understanding of direct and inverse proportion to solve problems. W

- Explain a method for determining whether a real-world problem involves direct proportion or inverse proportion. [SP, CU, MC]
- Explain a method for solving a real-world problem involving direct proportion. [CU, MC]
- Explain a method for solving a real-world problem involving inverse proportion. [CU, MC]
- Solve problems using direct or inverse models (e.g., similarity, age of car vs. worth). [SP, MC]
- Explain, illustrate, or describe examples of direct proportion. [CU]
- Explain, illustrate, or describe examples of inverse proportion. [CU]
- Use direct or inverse proportion to determine a number of objects or a measurement in a given situation.

Computation

1.1.6 Apply strategies to compute fluently with rational numbers in all forms including whole number exponents. W

- Complete multi-step computations using order of operations in situations involving combinations of rational numbers including whole number exponents and square roots of square numbers. [MC]
- Calculate using order of operations on all forms of rational numbers (e.g., $(3 \cdot 2 + 5)2 - 8$, $22 + 32$).
- Use properties to reorder and rearrange expressions to compute more efficiently. [RL]

Estimation

1.1.8 Apply estimation strategies to determine the reasonableness of results in situations involving multi-step computations with rational numbers including whole number powers and square and cube roots. W

- Identify when an approximation is appropriate. [MC]
- Explain situations involving real numbers where estimates are sufficient and others for which exact value is required. [CU]
- Justify why an estimate would be used rather than an exact answer in a given situation. [CU]
- Describe various strategies used during estimation involving integers, rational numbers. [CU]
- Use estimation to predict or to verify the reasonableness of calculated results. [RL]

Component 1.2: Understand and apply concepts and procedures from measurement.

Attributes, units, and systems

1.2.1 Analyze how changes in one or two dimensions of an object affect perimeter, area, surface area, and volume. W

- Describe and compare the impact that a change in one or more dimensions has on objects (e.g., how doubling one dimension of a cube affects the surface area and volume). [CU, MC]
- Describe how changes in the dimensions of objects affect perimeter, area, and volume in real world situations (e.g., how does the change in the diameter of an oil drum affect the area and volume). [CU, MC]
- Solve problems by deriving the changes in two dimensions necessary to obtain a desired surface area and/or volume (e.g., given a box with certain dimensions, make the volume of the box y cubic units by changing two dimensions of the box). [SP]
- Compare a given change in one or two dimensions on the perimeter, area, surface areas, or volumes of two objects.
- Determine the change in one dimension given a change in perimeter, area, volume, or surface area.

1.2.3 Understand how to convert units of measure within systems (U.S. or metric). W

- Understand how to convert units of measure within U.S. or within metric systems to achieve an appropriate level of precision.
- Convert within a system to a unit size appropriate to a given situation.
- Convert to a larger unit within a system while maintaining the same level of precision (e.g., represent 532 centimeters to 5.32 meters).
- Convert to a smaller unit within a system to increase the precision of a derived unit of measurement.

Procedures, precision, and estimation

1.2.5 Apply formulas to calculate measurements of right prisms or right circular cylinders. W

- Explain how to use a formula for finding the volume of a prism or cylinder. [CU, MC]
- Use a formula to find the volume of a prism or cylinder. [RL, MC]
- Use a formula to derive a dimension of a right prism or right cylinder given other measures.

- Use formulas to describe and compare the surface areas and volumes of two or more right prisms and/or right cylinders. [RL]
- Use formulas to obtain measurements needed to describe a right cylinder or right prism.

1.2.6 Understand and apply strategies to obtain reasonable measurements at an appropriate level of precision. W

- Identify situations in which approximate measurements are sufficient.
- Estimate a reasonable measurement at an appropriate level of precision. [MC]
- Estimate quantities using derived units of measure (e.g., distance or time using miles per hour, cost using unit cost). [MC]
- Estimate derived units of measure (e.g., miles per hour, people/year, grams/cubic centimeters). [MC]
- Apply a process that can be used to find a reasonable estimate for the volume of prisms, pyramids, cylinders, and cones.
- Estimate volume and surface area for right cylinders and right prisms.

Component 1.3: Understand and apply concepts and procedures from geometric sense.

Properties and relationships

1.3.1 Understand the relationship among characteristics of one-dimensional, two-dimensional, and three-dimensional figures. W

- Identify and label one- and two-dimensional characteristics (rays, lines, end points, line segments, vertices, and angles) in three-dimensional figures. [CU]
- Match or draw three-dimensional objects from different perspectives using the same properties and relationships (e.g., match to the correct net, draw the top view). [RL]
- Draw and label with names and symbols nets of right prisms and right cylinders. [RL, CU]
- Describe everyday objects in terms of their geometric characteristics. [CU]
- Describe or classify various shapes based on their characteristics.
- Make and test conjectures about two-dimensional and three-dimensional shapes and their individual attributes and relationships using physical, symbolic, and technological models (e.g., diagonal of a rectangle or prism is the longest interior segment; what figures make up cross-sections of a given three-dimensional shape). [SP, RL, CU, MC]

1.3.2 Apply understanding of geometric properties and relationships. W

- Use geometric properties and relationships to describe, compare, and draw two-dimensional and three-dimensional shapes and figures.
- Construct geometric figures using a variety of tools and technologies (e.g., angle bisectors, perpendicular bisectors, triangles given specific characteristics). [MC]
- Draw a plane shape and justify the answer given a set of characteristics. [RL, CU]
- Use the properties of two-dimensional and three-dimensional shapes to solve mathematical problems (e.g., find the width of a river based on similar triangles; given a set of parallel lines, a transversal, and an angle, find the other angles). [SP, RL, CU, MC]
- Compare two-dimensional and three-dimensional shapes according to characteristics including faces, edges, and vertices, using actual and virtual modeling. [RL, CU]

- Use technology to generate two and three dimensional models of geometric figures with given geometric characteristics (e.g., generate a two-dimensional animation using pentagons with fixed coordinates for one edge). [RL, SP]
- Create a three-dimensional scale drawing with particular geometric characteristics. [SP, CU, MC]

Locations and transformations

1.3.3 Apply understanding of geometric properties and location of points to figures. W

- Use coordinates to describe or identify the location of objects on coordinate grids.
- Describe geometric characteristics of two-dimensional objects using coordinates on a grid. [MC]
- Describe the location of points that satisfy given conditions (e.g., the set of points equidistant from a given point; a point equidistant from a given set of points). [CU]
- Represent situations on a coordinate grid or describe the location of points that satisfy given conditions (e.g., locate a gas station to be equidistant from given cities; locate a staking point to maximize the grazing area of a tethered goat). [MC, SP, RL]
- Use tools and technology to draw objects on a coordinate grid based on given conditions. [CU]
- Identify, interpret, and use the meaning of slope of a line as a rate of change using physical, symbolic, and technological models. [SP, RL, MC]

1.3.4 Apply understanding of multiple transformations. W

- Apply multiple transformations to create congruent and similar figures in any or all of the four quadrants.
- Use multiple transformations (combinations of translations, reflections, or rotations) to draw an image. [RL]
- Use dilation (expansion or contraction) of a given shape to form a similar shape. [RL, CU]
- Determine the final coordinates of a point after a series of transformations. [RL, CU]
- Examine figures to determine rotational symmetry about the center of the shape. [RL, MC]
- Define a set of transformations that would map one onto the other given two similar shapes. [SP, RL]
- Create a design with or without technology using a combination of two or more transformations with one or two two-dimensional figures. [SP, RL]
- Use technology to create two- and three-dimensional animations using combinations of transformations. [MC, SP, RL]

Component 1.4: Understand and apply concepts and procedures from probability and statistics.

Probability

1.4.1 Understand the concept of conditional probability. W

- Compare the probabilities of dependent and independent events. [CU, MC]
- Determine and justify whether the outcome of a first event affects the probability of a later event (e.g., drawing cards from a deck with or without replacement). [CU]
- Explain the difference between dependent and independent events. [CU]
- Explain and give examples of compound events. [CU]

1.4.2 Apply understanding of dependent and independent events to calculate probabilities.

W

- Determine probabilities of dependent and independent events. [SP]
- Generate the outcomes and probability of multiple independent and dependent events using a model or procedure (e.g., tree diagram, area model, counting procedures).
- Generate the outcomes and probability of events using a counting procedure (e.g., the number of license plates that can be made with three letters and three numbers; winning the lottery). [MC]
- Explain the relationship between theoretical probability and empirical frequency of dependent events using simulations with and without technology. [CU]
- Create a simple game based on independent probabilities wherein all players have an equal probability of winning. [MC, SP]
- Create a simple game based on compound probabilities. [MC, SP]
- Determine the sample space for independent or dependent events.

Statistics

1.4.3 Apply appropriate methods and technology to collect data or evaluate methods used by others for a given research questions. **W**

- Identify sources of bias in data collection questions, samples, and/or methods and describe how such bias can be controlled. [RL, CU]
- Evaluate methods and technology used to investigate a research question. [CU, MC]
- Collect data using appropriate methods.
- Use technology appropriately to collect data. [RL, MC]
- Identify appropriate data collection methods that might impact the accuracy of the results of a given situation. [RL, CU]
- Determine whether the sample for a given study was from a representative sample.
- Determine whether the methods of data collection used were appropriate for a given question or population. [RL]

1.4.4 Understand and apply techniques to find the equation for a reasonable linear model.

W

- Determine the equation for a reasonable line to describe a set of bivariate data. [RL, MC]
- Determine the equation of a line that fits the data displayed on a scatter plot. [SP, RL]
- Use technology to determine the line of best fit for a set of data. [MC]
- Match an equation with a set of data. [MC]
- Match an equation with a graphic display. [MC]
- Create a graph based on the equation for the line.

1.4.5 Analyze a linear model to judge its appropriateness for a data set. **W**

- Determine whether a straight line is an appropriate way to describe a trend in a set of bivariate data. [MC, RL]
- Determine whether the underlying model for a set of data is linear. [RL, MC]
- Decide and explain whether it is appropriate to extend a given data set following a line of best fit. [RL, MC]

- Determine whether a linear prediction from a given set of data is appropriate for the data and support the decision with data. [MC].
- Determine whether an equation for a line is appropriate for a given set of data and support the judgment with data. [RL, MC]
- Use technology to generate data to fit a linear model. [SP, MC]

1.4.6 Apply understanding of statistics to make, analyze, or evaluate a statistical argument.

W

- Identify trends in a set of data in order to make a prediction based on the information. [CU, MC]
- Justify a prediction or an inference based on a set of data. [CU, MC]
- State possible factors that may influence a trend but not be reflected in the data (e.g., population growth of deer vs. availability of natural resources or hunting permits). [MC, CU, RL]
- Use statistics to support different points of view. [RL]
- Analyze a set of statistics to develop a logical point of view. [RL, CU, MC]
- Justify or refute claims and supporting arguments based on data. [CU, MC]
- Determine whether statistics have been used or misused to support a point of view or argument and support the evaluation with data. [RL]

Component 1.5: Understand and apply concepts and procedures from algebraic sense.
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Patterns, functions, and other relations

1.5.1 Apply processes that use repeated addition (linear) or repeated multiplication (exponential). **W**

- Recognize, extend, or create a pattern or sequence between sets of numbers and/or linear patterns. [RL, CU, MC]
- Identify, extend, or create a geometric or arithmetic sequence or pattern. [RL, CU]
- Translate among equivalent numerical, graphical, and algebraic forms of a linear function. [RL, MC]
- Make predictions based on a pattern or sequence.

1.5.2 Analyze a pattern, table, graph, or model involving repeated addition (linear) or repeated multiplication (exponential) model to write an equation or rule. **W**

- Find the equation of a line in a variety of ways (e.g., from a table, graph, slope-intercept, point-slope, two points). [RL, MC]
- Generate and use rules for a pattern to make predictions about future events (e.g., population growth, future sales, growth of corn stalks, future value of savings account). [SP, RL, MC]
- Identify or write an equation or rule to describe a pattern, sequence, and/or a linear function. [RL, CU, MC]
- Write an equation for a line given a set of information (e.g., two points, point-slope, etc.). [CU, MC]
- Write a recursive definition of a geometric pattern (e.g., Start and New = Old * Number). [CU, MC]
- Represent systems of equations and inequalities graphically. [RL, MC]

- Write a story that represents a given linear equation or expression. [CU, MC]
- Write an expression, equation, or inequality with two variables representing a linear model of a real-world problem. [CU, MC]

Symbols and representations

1.5.4 Apply understanding of equations, tables, or graphs to represent situations involving relationships that can be written as repeated addition (linear) or repeated multiplication (exponential). W

- Represent variable quantities through expressions, equations, inequalities, graphs, and tables to represent linear situations involving whole number powers and square and cube roots. [CU, MC]
- Identify and use variable quantities to read and write expressions and equations to represent situations that can be described using repeated addition (e.g., models that are linear in nature). [CU, MC]
- Identify and use variable quantities to read and write expressions and equations to represent situations that can be described using repeated multiplication (e.g., models that are exponential such as savings accounts and early stages of population growth). [CU, MC]
- Recognize and write equations in recursive form for additive models (e.g., starting value, $\text{New} = \text{Old} + \text{some number}$). [CU, MC]
- Recognize and write equations in recursive form for multiplicative models (e.g., starting value, $\text{New} = \text{Old} \times \text{some number}$). [CU, MC]
- Select an expression or equation to represent a given real world situation. [MC]

Evaluating and solving

1.5.5 Apply procedures to simplify expressions. W

- Simplify expressions and evaluate formulas involving exponents.
- Justify a simplification of an expression involving exponents. [RL, CU]
- Use multiple mathematical strategies and properties to simplify expressions.

1.5.6 Apply procedures to solve equations and systems of equations. W

- Rearrange formulas to solve for a particular variable (e.g., given $A = .5bh$, solve for h). [MC, CU]
- Solve real-world situations involving linear relationships and verify that the solution makes sense in relation to the problem. [SP, RL, CU, MC]
- Find the solution to a system of linear equations using tables, graphs, and symbols. [CU, MC]
- Interpret solutions of systems of equations. [CU, MC]
- Solve multi-step equations. [SP, RL]
- Use systems of equations to analyze and solve real-life problems. [SP, CU, MC]
- Determine when two linear options yield the same outcome (e.g., given two different investment or profit options, determine when both options will yield the same result).
- Use systems of equations to determine the most advantageous outcome given a situation (e.g., given two investment options, determine under what conditions each will yield the best result.). [MC, SP]

EALR 2: The student uses mathematics to define and solve problems.

Component 2.1: Investigate situations.

Example: The following are the times (in seconds) of the Olympics in the given years. Using this information, is it reasonable to believe that the women will run as fast as the men in this event?

Justify your answer using this data:

<i>Year</i>	<i>Men's</i>	<i>Women's</i>	<i>Year</i>	<i>Men's</i>	<i>Women's</i>
1948	10.3	11.9	1976	10.06	11.08
1952	10.4	11.5	1980	10.25	11.06
1956	10.5	11.5	1984	9.99	10.97
1960	10.2	11.0	1988	9.92	10.54
1964	10	11.4	1992	9.96	10.82
1968	9.95	11.0	1996	9.84	10.94
1972	10.14	11.07	2000	9.87	10.75

2.1.1 Analyze a situation to define a problem. W

- Use strategies to become informed about the situation (e.g., listing information; examine the table for patterns; create a scatter plot to look for patterns; asking questions).
- Summarize the problem (e.g., there are Olympic winning times over the past 50 years; both men's and women's times are decreasing; will there come a time when women run faster than men).
- Determine whether enough information is given to find a solution (e.g., list what is needed to be found; extend the pattern to see if women's times will be less).
- Determine whether information is missing or extraneous (e.g., compare the list of known things to the list of needed things to see if there are things that are not needed).
- Define the problem (e.g., if the pattern continues in the same fashion, will women run faster than men and, if so, when will that occur).

Component 2.2: Apply strategies to construct solutions.**2.2.1 Apply strategies, concepts, and procedures to devise a plan to solve the problem. W**

- Organize relevant information from multiple sources (e.g., create a list of known and unknown information; create a scatter plot of men's and women's times vs. time on the same coordinate axis to analyze the patterns).
- Select and apply appropriate mathematical tools to devise a strategy in a situation (e.g., if the data, in either tabular or graphical form, suggest a linear relationship, plan to find a linear equation for each set of data; solve those equations simultaneously [or use technology to find the intersection of the two lines] to answer the question). If the data pattern suggests a non-linear model, plan to project what the pattern is and extend that pattern.

2.2.2 Apply mathematical tools to solve the problem. W

- Implement the plan devised to solve the problem (e.g., solve the set of simultaneous equations to arrive at a time where the two times are the same).
- Use mathematics to solve the problem (e.g., use algebra to write equations for the two linear models, solve the system of equations using either symbols or technology).

- Identify when an approach is unproductive and modify or try a new approach (e.g., if the result does not make sense in the context, return to the plan to see if something has gone wrong and adjust accordingly).
- Check the solution to see if it works (e.g., the solution may be a partial year [i.e., 2003.6]; decide how to deal with this and also if the year is reasonable [i.e., 1925 does not make sense given the context]).

EALR 3: The student uses mathematical reasoning.

Component 3.1: Analyze information.

3.1.1. Synthesize information from multiple sources in order to answer questions. W

- Use the properties of two-dimensional and three-dimensional figures to solve mathematical problems (e.g., find the width of a river based on similar triangles; given a set of parallel lines, a transversal, and an angle, find the other angles).

Component 3.2: Make predictions, inferences, conjectures, and draw conclusions.

3.2.1 Apply skill of conjecturing and analyze conjectures by formulating a proof or constructing a counter example. W

- Make and test conjectures about two-dimensional and three-dimensional figures and their individual attributes and relationships using physical, symbolic, and technological models (e.g., diagonal of a rectangle or prism is the longest interior segment; what figures make up cross-sections of a given three-dimensional shape). (1.3.1)

3.2.2 Analyze information to draw conclusions and support them using inductive and deductive reasoning. W

- Compare and describe the volume of cylinders, cones, and prisms when an attribute is changed (e.g., the area of the base, the height of solid). (1.2.4)
- Draw a plane shape of a given set of characteristics and justify the answer. (1.3.2)
- Identify trends in a set of data in order to make a prediction based on the information. (1.4.6)
- Use statistics to support different points of view. (1.4.6)

3.2.3 Analyze procedures to determine appropriateness of claims and arguments. W

- Examine claims and supporting arguments based on data and make needed revisions. (1.4.6)

Component 3.3: Verify results.

3.3.1 Analyze results using inductive and deductive reasoning.

- Compare and contrast similar two-dimensional figures and shapes using properties of two-dimensional figures and shapes. (1.3.2)
- Find a reasonable estimate for the volume of prisms, pyramids, cylinders, and cones. (1.2.6)

3.3.2 Analyze thinking and mathematical ideas using models, known facts, patterns, relationships, counter examples, or proportional reasoning. W

- Examine a set of data, research other sources to see if the data is consistent, find articles written to see if the data makes sense, to develop a logical point of view and to support that view. (1.4.6)

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

Component 4.1: Gather information.

4.1.1 Understand how to develop or apply an efficient system for collecting mathematical information for a given purpose. W

- Collect data efficiently on the outcomes of first events and later events to determine and justify how the first event affects the probability of later events (e.g., drawing cards from a deck with or without replacement). (1.4.1)

4.1.2 Synthesize mathematical information for a given purpose from multiple, self-selected sources. W

- State possible factors that may influence a trend but not be reflected in the data (e.g., population growth of deer vs. availability of natural resources or hunting permits). (1.4.6)

Component 4.2: Organize, represent, and share information.

4.2.1 Analyze mathematical information to organize, clarify, and refine an argument. W

- Develop an argument to support a given point of view and set of statistics. (1.4.6)

4.2.2 Understand how to express ideas and situations using mathematical language and notation. W

- Explain how division of measurements produces a derived unit of measurement (e.g., miles traveled divided by hours traveled yields the derived unit [miles per hour]). (1.2.2)
- Describe the location of points that satisfy given conditions (e.g., the set of points equidistant from a given point; a point equidistant from a given set of points). (1.3.3)
- Describe and compare the impact that a change in one or more dimensions has on objects (e.g., doubling the edge of a cube affects the surface area). (1.2.1)
- Explain the relationship between theoretical probability and empirical frequency of dependent events using simulations with and without technology. (1.4.2)

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

Component 5.1: Relate concepts and procedures within mathematics.

5.1.1 Apply multiple mathematical concepts and procedures in a given problem or situation. [W]

- Estimate derived units of measure (e.g., miles per hour, people/year, grams/cubic centimeters). (1.2.6)
- Determine the final coordinates of a point after a series of transformations. (1.3.4)

5.1.2 Understand how use different mathematical models and representations in the same situation. [W]

- Identify, interpret, and use the meaning of slope of a line as a rate of change using concrete, symbolic, and technological models. (1.2.2)
- Construct one-dimensional, two-dimensional, and three-dimensional geometric figures using a variety of tools and technologies (e.g., angle bisectors, perpendicular bisectors, triangles given specific characteristics). (1.3.2)
- Find the equation of a line in a variety of ways (e.g., from a table, graph, slope-intercept, point-slope, two points). (1.5.1)
- Find the solution to a system of linear equations using tables, graphs and symbols. (1.5.6)

5.2: Relate mathematical concepts procedures to other disciplines.

5.2.1 Analyze mathematical patterns and ideas to extend mathematical thinking and modeling in other disciplines.

- Justify a prediction or an inference based on a set of data. (1.4.6)
- Create a physical activity plan that results in a specified number of calories over a specified time. [PE]

5.2.2 Know contributions of individuals and cultures to the development of mathematics.

- Recognize the mathematical contribution of a person or culture (e.g., create a report or presentation that highlights a mathematical contribution related to current mathematical study).

Component 5.3: Relate mathematical concepts and procedures to real-world situations.

5.3.1 Understand situations in which mathematics can be used to solve problems with local, national, or international implications.

- Explain a method for determining whether a real world problem involves direct proportion or inverse proportion. (1.1.4)
- Describe how changes in the dimensions of objects affect perimeter, area, and volume in real-world situations (e.g., how does the change in the diameter of an oil drum affect the area and volume). (1.2.1)
- Represent situations on a coordinate grid or describe the location of points that satisfy given conditions (e.g., locate a gas station to be equidistant from given cities; locate a staking point to maximize the grazing area of a tethered goat). (1.3.3)

5.3.2 Understand the mathematical knowledge and training requirements for occupational/career areas of interest.

- Select a career and research the mathematics necessary to get the job and the mathematics used in the job.