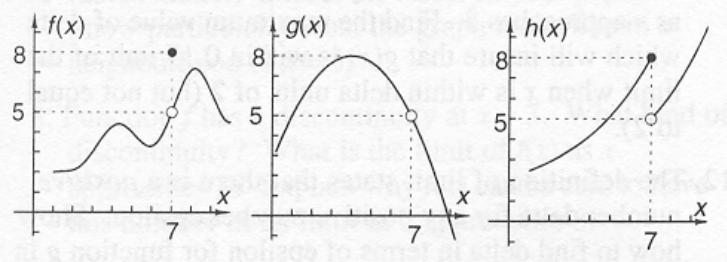
Calculus Practice Test

Name_____

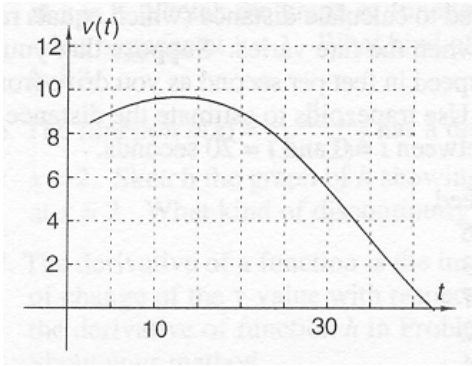
Period_____

Directions: Solve the following problems. Show all your work.

1. Below are the graphs of three functions, f, g, and h. Explain why f(x) and g(x) have limits as x approaches 7, but h(x) does not have a limit.



2. A concept of calculus is the definite integral, which is a means of finding the product of x and y in a function where y varies. Definite integrals can be calculated graphically. The following graph shows the velocity v(t), in feet per second, of a moving object. (a)Find an estimate of the definite integral of v(t) with respect to t from t = 10 to t = 30 seconds. (b)What is the physical meaning of the answer in part a?



3. A concept of calculus is the derivative, which is an instantaneous rate of change. For the graph above, at t = 20 seconds, approximately what is the instantaneous rate of change of velocity? Is the velocity increasing or decreasing? Explain.

4. Derivatives and definite integrals can be estimated from tables of data. The following data shows the heat capacity, C(T) British thermal units (Btu) per degree, of a pound of steam at various Fahrenheit temperatures.

(a) Use the trapezoidal rule to estimate the definite integral of C(T) with respect to T from T = 1000° to T = 3000°.

T	C(T)
1000	9.2
1500	10.1
2000	10.8
2500	11.4
3000	12.0

(b) What are the units of the integral in part a? What does it represent in the real world?

(c) Approximately what is the rate of change of heat capacity in the data at T = 2000°?

5. Calvin and Phoebe start up a hill in their car. When they are 50 feet from the bottom of the hill the car runs out of gas. They coast to a stop then start rolling backwards. Phoebe figures that their displacement, D(t), from the bottom of the hill is given by

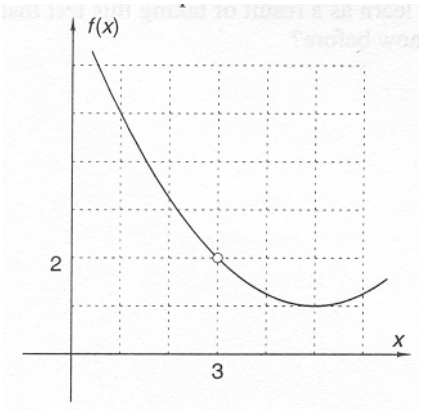
$$D(t) = -2t^2 + 20t + 50.$$

(a) How far are they from the bottom of the hill when t = 3 seconds?

(b) Find Calvin and Phoebe's average velocity for the interval t = 3 to t = 3.1 seconds.

(c) By using times closer and closer to t = 3, make a conjecture about the limit the average velocity approaches as t approaches 3.

6. Function f, below, has 2 as its limit as x approaches 3. If epsilon in the definition of limit is 1, estimate the maximum possible value of delta.



7. Does function f in problem 6 have a limit as x approaches 1? If so estimate the limit. If not, tell why not.

8. Definite integrals are products of the x and y variables of a function. For instance, a definite integral is used to calculate distance (which equals rate times time) when the rate varies. Suppose that you record your speed in feet per second as you drive from a parking lot. Use trapezoids to estimate the distance you travel between t = 0 and t = 20 seconds.

t	speed
0	6
5	13
10	17
15	22
20	14

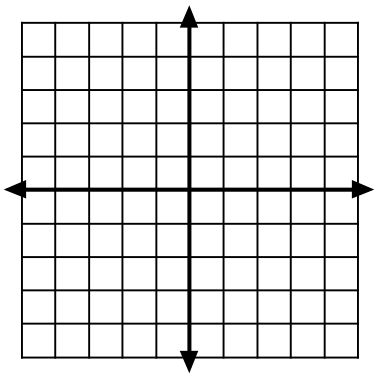
9. As you later cruise down the highway, your speed in feet per second is given by

$$v(t) = 30 + 0.3\sqrt{t}$$

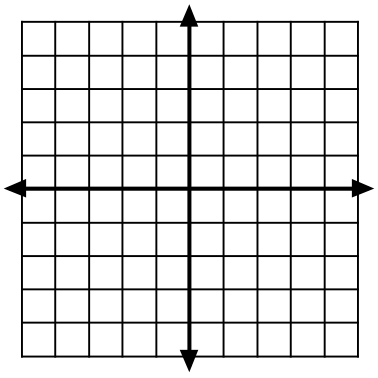
Use your trapezoid rule program to estimate the distance you go between t = 0 and t = 25 seconds using n = 10 increments and using n = 100 increments. What integer value does your distance seem to be approaching as a limit as n gets larger?

10. Write the definition of limit.

11. The function $g(x) = 2^x \cdot \frac{|x-1|}{x-1}$ has a discontinuity at x = 1. Sketch the graph of g showing what happens at x = 1. What kind of discontinuity is it?



12. The function $h(x) = \frac{1}{(x-2)^2}$ has a discontinuity at x = 2. Sketch the graph of h showing what happens at x = 2. What kind of discontinuity is it?



Directions: For problems 13 and 14, the table shows the force (pounds) needed to stretch a bungee cord to a certain length (feet). The amount of work done in stretching a bungee cord equals the force exerted on the cord times the distance it stretches.

Feet	Pounds
10	17
12	20
14	24
16	30
18	37
20	48

13. Perform an appropriate computation to find the amount of work done in stretching the cord from 10 feet to 20 feet. Name the method used. Tell the units of work in this instance.

14. Use a symmetric difference quotient to estimate the instantaneous rate of change of force with respect to distance at 16 feet. What units does this rate have?