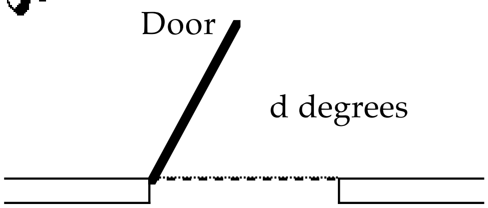
Exploration 1

Instantaneous Rate of Change of a Function

Name______________

Period______



The diagram shows a door with an automatic closer. At time t = 0 seconds someone pushes the door. It swings open, slows down, stops, starts closing, then slams shut at time t = 7 seconds. As the door is in motion the number of degrees, d, it is from its closed position depends on t.

1. Sketch a reasonable graph of d versus t.

2. Suppose that d is given by the equation
$d = 200t \cdot 2^{-t}$.
Plot this graph on your calculator. Sketch the results here.

3. Make a table of values of d for each second from t = 0 through t = 10. Round to the nearest 0.1°.

t	d	t	d
0		6	
1		7	
2		8	
3		9	
4		10	
5			

4. At time t = 1 second, does the door appear to be opening or closing? How do you tell?

5. What is the average rate at which the door is moving for the time interval [1, 1.1]? Based on your answer, does the door seem to be opening or closing at time t = 1? Explain.

6. Find an estimate of the instantaneous rate at which the door is moving at time t = 1 second. Show how you get your answer.

7. In calculus you will learn by four methods:
• algebraically,
• numerically,
• graphically,
• verbally (talking and writing).
What did you learn as a result of doing this Exploration that you did not know before?

$\int \sin x e^{\cos x} dx$ Pre-Calculus

Exploration 1 Assignment

Name________________

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1. Pendulum Problem: A pendulum hangs from the ceiling. As the pendulum swings, its distance, d cm, from one wall of the room depends on the number of seconds, t, since it was set in motion. Assume that the equation for d as a function of t is

$$d = 80 + 30\cos\frac{\pi}{3}t, \qquad t \geq 0.$$

It is desired to find out how fast the pendulum is moving at a given instant, t, and whether it is approaching or going away from the wall.

a. Find d when t = 5. If you don't get 95 for the answer, make sure your calculator is in radian mode.

b. Estimate the instantaneous rate of change of d at t = 5 by finding the average rates for t = 5 to 5.1, t = 5 to 5.01, and t = 5 to 5.001.

c. Why can't the actual instantaneous rate of change of d with respect to t be calculated using the method in 1b?

d. Estimate the instantaneous rate of change of d with resect to t at t = 1.5. At that time is the pendulum approaching the wall or going away from it? Explain.

e. How is the instantaneous rate of changed related to the average rates? What name is given to the instantaneous rate?

f. What is the reason for the domain restriction t ≥ 0? Can you think of any reason that there would be an upper bound to the domain?

2. Board Price Problem: If you check the prices of various lengths of lumber, you will find that a board twice as long as another of the same type does not necessarily cost twice as much. Let x be the number of feet long a 2″ x 6″ board is and let y be the number of cents you pay for the board. Assume that y is given by

$$y = 0.2x^3 - 4.8x^2 + 80x$$

a. Find the price of 2″ x 6″ boards that are 5 ft long, 10 ft long, and 20 ft long.

b. Find the average rate of change of the price in cents per foot for 5 ft to 5.1 ft, 5 ft to 5.01 ft, and 5ft to 5.001 ft.

c. The average number of cents per foot in 2b is approaching an integer as the change in x gets smaller and smaller. What integer? What is the name given to this rate of change?

d. Estimate the instantaneous rate of change in price if x is 10 ft and if x is 20 ft. You should find that each of these rates is an integer.

e. One of the principles of marketing is that when you buy in larger quantities, you usually pay less per unit. Explain how the numbers in Problem 2 show that this principle does not apply to buying longer boards. Think of a reason why it does not apply.