

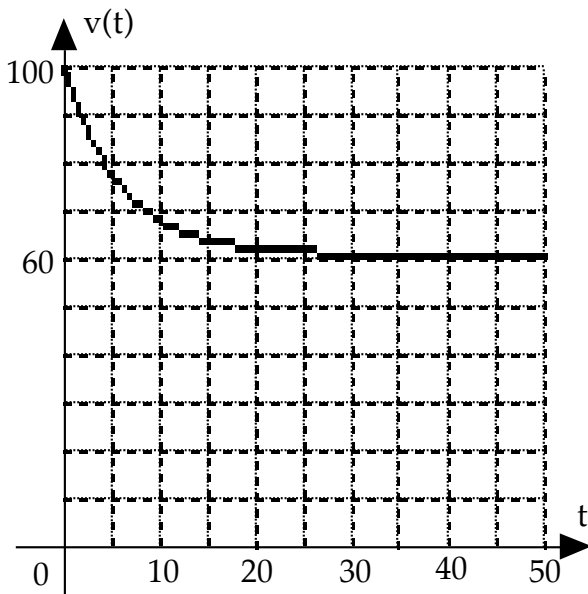
Exploration 3

Introduction to Definite Integrals

Name _____

Period _____

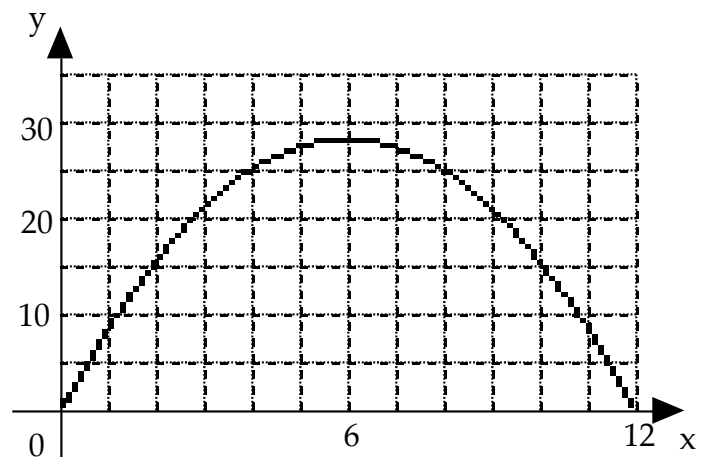
As you drive on the highway you accelerate to 100 feet per second to pass a truck. After you have passed, you slow down to a more moderate 60 ft/sec. The diagram shows the graph of your velocity, $v(t)$, as a function of the number of seconds, t , since you started slowing.



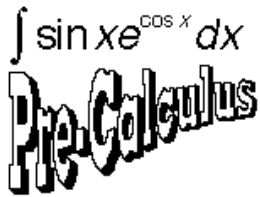
1. What does your velocity seem to be between $t = 30$ and $t = 50$ seconds? How far do you travel in the time interval $[30, 50]$?
2. Explain why the answer to Problem 1 can be represented as the area of a rectangle region of the graph. Shade this region.
3. The distance you travel between $t = 0$ and $t = 20$ can also be represented as the area of a region bounded by the (curved) graph. Count the number of squares in this region. Estimate the area of parts of squares to the nearest 0.1 square space. For instance, how would you count this partial square?



4. How many feet does each small square on the graph represent? How far, therefore, did you go in the time interval $[0, 20]$?
5. Problem 3 and 4 involved finding the product of the x -value and the y -value for a function where y may vary with x . Such a product is called the definite integral of y with respect to x . Based on the units of t and $v(t)$, explain why the definite integral of $v(t)$ with respect to t in Problem 4 has feet for its units.
6. The graph shows the cross-sectional area, y square inches, of a football as a function of the distance, x inches, from one of its ends. Estimate the definite integral of y with respect to x .



7. What are the units of the definite integral in Problem 6? What, therefore, do you suppose the definite integral represents?
8. What did you learn as a result of doing this Exploration that you did not know before?



Exploration 3 Assignment

Name _____

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For Problems 1 - 4, estimate the definite integral by counting squares on a graph.

1. $f(x) = -0.1x^2 + 7$ $x = 0$ to $x = 5$

2. $f(x) = -0.2x^2 + 8$ $x = -2$ to $x = 5$

3. $h(x) = \sin x$ $x = 0$ to $x = \pi$

4. $g(x) = 2^x + 5$ $x = -1$ to $x = 1$

For Problems 5 - 6, estimate the derivative of the function at the given value of x .

5. $f(x) = \tan x$, $x = 1$

6. $h(x) = -7x + 100$, $x = 5$

7. Sports Car Problem: You have been hired by an automobile manufacturer to analyze the predicted motion of a new sports car they are building. When accelerated hard from a standing start, the velocity of the car, $v(t)$ ft/sec, is expected to vary exponentially with time, t seconds, according to the equation

$$v(t) = 100(1 - 0.9^t).$$

- Draw the graph of the function v in the domain $[0, 10]$.
- What is the range of the velocity function? range for the domain?
- Approximately how many seconds will it take the car to reach 60 ft/sec?
- Approximately how far will the car have traveled when it reaches 60 ft/sec?
- At approximately what rate is the velocity changing when $t = 5$?
- What special name is given to the rate of change of velocity?

8. Slide Problem: Phoebe sits atop the swimming pool slide. At time $t = 0$ sec she pushes off. Calvin ascertains that her velocity, $v(t)$, is given by

$$v(t) = 10\sin 0.3t.$$

where $v(t)$ is in feet per second. Phoebe splashes into the water at time $t = 4$ sec.

- a. Plot the graph of function v . (Don't forget to set your calculator to radian mode.)
- b. What are the domain and range of the velocity function?
- c. How fast was she going when she hit the water?
- d. Approximately how long is the slide?
- e. At approximately what rate was her velocity changing at $t = 3$?
- f. What special name is given to the rate of change of velocity?