

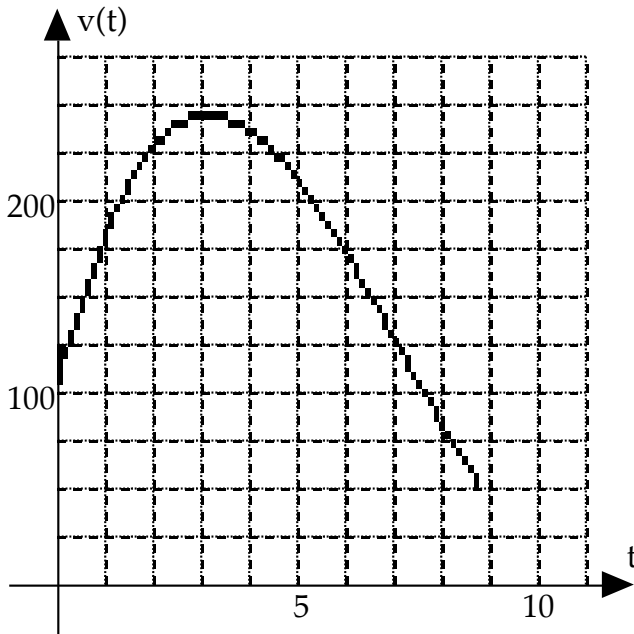
# Exploration 4

## Definite Integrals by Trapezoidal Rule

Name \_\_\_\_\_

Period \_\_\_\_\_

**Rocket Problem:** Ella Vader (Darth's daughter) is driving in her rocket ship. At time  $t = 0$  minutes she fires her rocket engine. The ship speeds up for a while, then slows down as Alderaan's gravity takes its effect. The graph of her velocity,  $v(t)$  miles per minute, is shown below.



1. What mathematical concept would be used to estimate the distance Ella goes between  $t = 0$  and  $t = 8$ ?
2. Estimate the distance in Problem 1 geometrically.
3. Ella figures that her velocity is given by
 
$$v(t) = t^3 - 21t^2 + 100t + 110$$

Plot this graph on your calculator. Does the graph confirm or refute what Ella figures? Tell how you arrive at your conclusion.

4. Divide the region under the graph from  $t = 0$  to  $t = 8$ , which represents the distance, into four vertical strips of equal width. Draw four trapezoids whose areas approximate the areas of these strips, and whose parallel sides extend from the x-axis to the graph. By finding the areas of these trapezoids, estimate the distance Ella goes. Does the answer agree with Problem 2?
5. The technique in Problem 4 is the trapezoidal rule. Put a program on our calculator to use this rule. The function equation may be stored as . Does the the programs answer agree with Problem 4.
6. Use the program from Problem 5 to estimate the definite integral using 20 trapezoids.
7. The exact value of the definite integral is the limit of estimate by trapezoids as the width of each trapezoids approaches zero. By using the program from Problem 5, make a conjecture about the exact value of the definite integral.
8. What is the fastest Ella went? At what time was that?
9. Approximately what was Ella's rate of change of velocity when  $t = 5$ ? Was she speeding up or slowing down at that time?
10. At what time does Ella stop? Based on the graph, does she stop abruptly or gradually?
11. What did you learn as a result of doing this Exploration that you did not know before?



# Exploration 4 Assignment

Name \_\_\_\_\_

Period \_\_\_\_\_

1. Spaceship Problem: A spaceship is being launched from Cape Canaveral. As the last stage of the rocket motor is firing, the velocity is given by

$$v(t) = (1600)(1.1^t)$$

where  $v(t)$  is in feet per second and  $t$  is the number of seconds since the last stage started.

- Plot the graph of  $v(t)$  versus  $t$ , from  $t = 0$  to  $t = 30$ . Sketch the result.
  - Tell why the area of the region under the graph represents the distance the spaceship went in this 30 sec interval.
  - Find, approximately, the distance traveled between  $t = 0$  and  $t = 30$  by using trapezoids of width corresponding to 5 sec. Sketch these trapezoids on your graph.
  - What mathematical term is used for the product of velocity and time found in this way?
  - To go into orbit around the earth, the spaceship must be going at least 27000 ft/sec. Will it be going this fast when  $t = 30$ ? Justify your answer.
2. Water over the Dam Problem: The amount of water that has flowed over the spillway on a dam can be estimated from the flow rate and the length of time the water has been flowing. Suppose that the flow rate been recorded every 3 hr for a 24 hr period, as shown in the table. Estimate the number of cubic feet of water that has flowed over the dam in this period.

time	ft <sup>3</sup> / hr	time	ft <sup>3</sup> / hr
12:00 am	5000	12:00 pm	11000
3:00 am	8000	3:00 pm	7000
6:00 am	12000	6:00 pm	4000
9:00 am	13000	9:00 pm	6000
		12:00 am	9000

3. Aircraft Carrier Landing Problem: In 1993, Kara Hultgreen became one of the first female pilots authorized to fly navy planes in combat. Assume that as she comes in for a landing on the carrier, her speed in feet per second takes on the values shown in the table. Find, approximately, how far her plane travels as it comes to a stop. Is her plane in danger of running off the other end of the 800 ft long flight deck?

sec	ft/sec
0.0	300
0.6	230
1.2	150
1.8	90
2.4	40
3.0	0

4. Integral as a Limit Problem: Now that you have a program to calculate definite integrals approximately, you can see what happens to the value of the integral as you use narrower trapezoids. Estimate the definite integral of  $f(x) = x^2$  from  $x = 1$  to  $x = 4$ , using 10, 100, and 1000 trapezoids. What number do the values seem to be approaching as the number of trapezoids gets larger and larger? Make a conjecture about the exact value of the definite integral as the width of each trapezoid approaches zero. This number is the \_\_\_\_\_ of the areas of the trapezoids as the limit of their widths approaches zero. What word goes in the blank?