

# Explorations 5

## Introduction to Limits

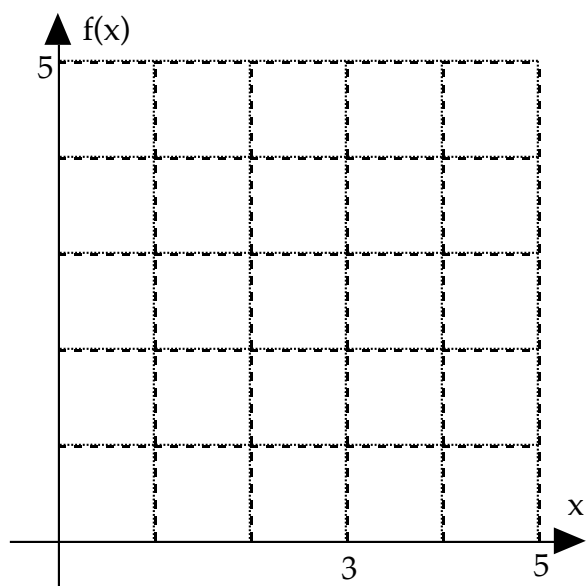
Name \_\_\_\_\_

Period \_\_\_\_\_

1. Plot on your calculator the graph of this function.

$$f(x) = \frac{x^3 - 7x^2 + 17x - 15}{x - 3}$$

Use a friendly window with  $x = 3$  as a grid point. Sketch the results here. Show the behavior of the function in a neighborhood of  $x = 3$ .



2. Substitute 3 for  $x$  in the equation for  $f(x)$ . What form does the answer take? What name is given to an expression of this form?
3. The graph of  $f$  has a removable discontinuity at  $x = 3$ . The  $y$ -value at this discontinuity is the limit of  $f(x)$  as  $x$  approaches 3. What number does this limit equal?
4. Make a table of values of  $f(x)$  for each 0.1 unit change in  $x$ -values from 2.5 through 3.5.

$x$	$f(x)$	3.0	
2.5		3.1	
2.6		3.2	
2.7		3.3	
2.8		3.4	
2.9		3.5	

5. Between what two numbers does  $f(x)$  stay when  $x$  is kept in the open interval  $(2.5, 3.5)$ ?
6. Simplify the fraction for  $f(x)$ . Solve numerically to find the two numbers close to 3 between which  $x$  must be kept if  $f(x)$  is to stay between 1.99 and 2.01.
7. How far from  $x = 3$  (to the left and to the right) are the two  $x$ -values in Problem 6?
8. For the statement "If  $x$  is within \_\_\_\_\_ units of 3 (but not equal to 3), then  $f(x)$  is within 0.01 unit of 2," write the largest number that can go in the blank.
9. Write the definition of limit.
10. Problem 8 gives four numbers that correspond to  $L$ ,  $c$ , epsilon, and delta in the definition of limit. Which is which?
11. What did you learn as a result of doing this Exploration that you did not know before?



# Exploration 5 Assignment

Name \_\_\_\_\_

Period \_\_\_\_\_

1. The function  $f(x) = \frac{4x^2 - 7x - 2}{x - 2}$  is undefined when  $x = 2$ . In this problem you will show that  $f(x)$  does have a limit as  $x$  approaches 2.
- Plot the graph of  $f$ . Use a friendly window that includes  $x = 2$  as a grid point. Sketch the graph and name the feature that seems to be present at  $x = 2$ .
  - From the graph, tell what you think the limit of  $f(x)$  is as  $x$  approaches 2.
  - Try to evaluate  $f(2)$  by direct substitution. What form does the answer take? What name is given to a form such as this?
  - Factor the numerator and simplify the expression by canceling the common factor. Although the simplified expression does not equal  $f(2)$ , you can substitute 2 for  $x$  and get an answer. What is this answer and what does it represent?
  - How close to 2 would you have to keep  $x$  in order for  $f(x)$  to be between 8.9 and 9.1?
  - How close to 2 would you have to keep  $x$  in order for  $f(x)$  to be within 0.001 unit of the limit in 1b? Answer in the form “ $x$  must be within \_\_\_\_\_ units of 2.”
  - Four constants appear in the definition of limit:  $L$ ,  $c$ , epsilon, and delta. What are the values of these four constants in 1f?
  - Explain how you could find a suitable value of delta no matter how small epsilon is.
  - What is the reason for the restriction “...but not equal to  $c$ ” that appears in the definition of limit?
2. The function  $f(x) = \frac{4x^2 - 7x - 2}{x - 2}$  is undefined when  $x = 2$ . However, if you cancel the  $(x - 2)$  factors, the equation becomes  $f(x) = x^2 - 6x + 13$  ( $x \neq 2$ ). So  $f$  would be a quadratic function, except that there is a removable discontinuity where  $x = 2$ . The  $y$ -value of this missing point is the limit of  $f(x)$  as  $x$  approaches 2.
- Show that  $f(2)$  has the indeterminate form  $0/0$ . Do an appropriate calculation to show that 5 is the limit of  $f(x)$  as  $x$  approaches 2.
  - Plot the graph close to the discontinuity. Use a friendly window that includes  $x = 2$  as a grid point and has an  $x$ -increment of 0.001. Then use your calculator to trace or the table feature to make a table of values of  $f(x)$  for each value of  $x$  from 1.990 through 2.010. For which values of  $x$  in the table is  $f(x)$  within 0.01 unit of 5? Complete the statement “If  $x$  is within \_\_\_\_\_ units of 2, then  $f(x)$  is within 0.01 unit of 5.”

- c. Find the largest interval of values of  $x$  for which  $f(x)$  is within 0.01 unit of 5. You can do this by setting  $f(x) = 4.99$  and solving to find the value of  $x$  nearest 2. Repeat for  $f(x) = 5.01$ . Keep as much precision as your calculator will give you.
- d. Sketch the part of the graph close to  $x = 2$ . Show how the numbers in 2c relate to the graph.
- e. Find the largest number you could put in the blank of the statement in 2b. Take into account that the interval in 2c is of a different width on one side of 2 than on the other.
- f. Write the values of the constants  $L$ ,  $c$ , epsilon, and delta for 2a-e.

3. One-Sided Limit Problem: The following function has a step discontinuity at  $x = 2$ .

$$f(x) = 3 + \frac{1}{2}x + \frac{|x - 2|}{x - 2}$$

- a. The figure below shows the graph of  $f$ . Explain why the graph takes a jump at  $x = 2$ .
- b. What is the limit of  $f(x)$  as  $x$  approaches 2 from the left side?
- c. What is the limit of  $f(x)$  as  $x$  approaches 2 from the right side?
- d. Explain why there is no single number that can be the limit of  $f(x)$  as  $x$  approaches 2.

